

**Objective** To use the Law of Detachment and the Law of Syllogism



Use each step to write an expression for the process as a whole.



Getting Ready!

You want to use the coupon to buy three different pairs of jeans. You have narrowed your choices to four pairs. The costs of the different pairs are \$24.99, \$39.99, \$40.99, and \$50.00. If you spend as little as possible, what is the average amount per pair of jeans that you will pay? Explain.

## BUY TWO PAIRS OF JEANS

### Get a **THIRD** Free\*

\*Free jeans must be of equal or lesser value.



### Lesson Vocabulary

- deductive reasoning
- Law of Detachment
- Law of Syllogism

In the Solve It, you drew a conclusion based on several facts. You used deductive reasoning. **Deductive reasoning** (sometimes called logical reasoning) is the process of reasoning logically from given statements or facts to a conclusion.

**Essential Understanding** Given true statements, you can use deductive reasoning to make a valid or true conclusion.

take note

### Property Law of Detachment

#### Law

If the **hypothesis** of a true conditional is true, then the **conclusion** is true.

#### Symbols

If  $p \rightarrow q$  is true  
and  $p$  is true,  
then  $q$  is true.

To use the Law of Detachment, identify the hypothesis of the given true conditional. If the second given statement matches the hypothesis of the conditional, then you can make a valid conclusion.



## Problem 1 Using the Law of Detachment

What can you conclude from the given true statements?

- A Given:** If a student gets an A on a final exam, then the student will pass the course.  
Felicia got an A on her history final exam.

If a student gets an A on a final exam, then the student will pass the course.  
Felicia got an A on her history final exam.

The second statement matches the hypothesis of the given conditional. By the Law of Detachment, you can make a conclusion.

**You conclude:** Felicia will pass her history course.

- B Given:** If a ray divides an angle into two congruent angles, then the ray is an angle bisector.

$\overrightarrow{RS}$  divides  $\angle ARB$  so that  $\angle ARS \cong \angle SRB$ .

If a ray divides an angle into two congruent angles, then the ray is an angle bisector.  
 $\overrightarrow{RS}$  divides  $\angle ARB$  so that  $\angle ARS \cong \angle SRB$ .

The second statement matches the hypothesis of the given conditional. By the Law of Detachment, you can make a conclusion.

**You conclude:**  $\overrightarrow{RS}$  is an angle bisector.

- C Given:** If two angles are adjacent, then they share a common vertex.  
 $\angle 1$  and  $\angle 2$  share a common vertex.

If two angles are adjacent, then they share a common vertex.  
 $\angle 1$  and  $\angle 2$  share a common vertex.

The information in the second statement about  $\angle 1$  and  $\angle 2$  does not tell you if the angles are adjacent. The second statement does not match the hypothesis of the given conditional, so you cannot use the Law of Detachment.  $\angle 1$  and  $\angle 2$  could be vertical angles, since vertical angles also share a common vertex. You cannot make a conclusion.

### Think

In part (C), the second statement is not a subset of the hypothesis. Instead, it is a subset of the conditional's conclusion.



- Got It?** 1. What can you conclude from the given information?
- If there is lightning, then it is not safe to be out in the open.  
Marla sees lightning from the soccer field.
  - If a figure is a square, then its sides have equal length.  
Figure  $ABCD$  has sides of equal length.

Another law of deductive reasoning is the Law of Syllogism. The **Law of Syllogism** allows you to state a conclusion from two true conditional statements when the conclusion of one statement is the hypothesis of the other statement.



### Property Law of Syllogism

#### Symbols

If  $p \rightarrow q$  is true  
and  $q \rightarrow r$  is true,  
then  $p \rightarrow r$  is true.

#### Example

If it is July, then you are on summer vacation.

If you are on summer vacation, then you work at a smoothie shop.

**You conclude:** If it is July, then you work at a smoothie shop.

## Plan

When can you use the Law of Syllogism?

You can use the Law of Syllogism when the conclusion of one statement is the hypothesis of the other.



### Problem 2 Using the Law of Syllogism

What can you conclude from the given information?

**A Given:** If a figure is a square, then the figure is a rectangle.  
If a figure is a rectangle, then the figure has four sides.

If a figure is a square, then the figure is a rectangle.

If a figure is a rectangle, then the figure has four sides.

The conclusion of the first statement is the hypothesis of the second statement, so you can use the Law of Syllogism to make a conclusion.

**You conclude:** If a figure is a square, then the figure has four sides.

**B Given:** If you do gymnastics, then you are flexible.  
If you do ballet, then you are flexible.

If you do gymnastics, then you are flexible.

If you do ballet, then you are flexible.

The statements have the same conclusion. Neither conclusion is the hypothesis of the other statement, so you cannot use the Law of Syllogism. You cannot make a conclusion.



- Got It?** 2. What can you conclude from the given information? What is your reasoning?
- If a whole number ends in 0, then it is divisible by 10.  
If a whole number is divisible by 10, then it is divisible by 5.
  - If  $\overrightarrow{AB}$  and  $\overrightarrow{AD}$  are opposite rays, then the two rays form a straight angle.  
If two rays are opposite rays, then the two rays form a straight angle.

You can use the Law of Syllogism and the Law of Detachment together to make conclusions.

### **Problem 3** Using the Laws of Syllogism and Detachment

What can you conclude from the given information?

**Given:** If you live in Accra, then you live in Ghana.

If you live in Ghana, then you live in Africa. Aissa lives in Accra.

If you live in Accra, then you live in Ghana.

If you live in Ghana, then you live in Africa.

Aissa lives in Accra.

You can use the first two statements and the Law of Syllogism to conclude:

If you live in Accra, then you live in Africa.

You can use this new conditional statement, the fact that Aissa lives in Accra, and the Law of Detachment to make a conclusion.

**You conclude:** Aissa lives in Africa.



#### Think

**Does the conclusion make sense?**

Accra is a city in Ghana, which is an African nation. So if a person lives in Accra, then that person lives in Africa. The conclusion makes sense.




**Got It?** 3. a. What can you conclude from the given information? What is your reasoning?

If a river is more than 4000 mi long, then it is longer than the Amazon.

If a river is longer than the Amazon, then it is the longest river in the world.

The Nile is 4132 mi long.

 b. **Reasoning** In Problem 3, does it matter whether you use the Law of Syllogism or the Law of Detachment first? Explain.




### Lesson Check

#### Do You Know HOW?


If possible, make a conclusion from the given true statements. What reasoning did you use?

- If it is Tuesday, then you will go bowling.  
You go bowling.
- If a figure is a three-sided polygon, then it is a triangle.  
Figure *ABC* is a three-sided polygon.
- If it is Saturday, then you walk to work.  
If you walk to work, then you wear sneakers.

#### Do You UNDERSTAND? MATHEMATICAL PRACTICES

 4. **Error Analysis** What is the error in the reasoning below?

~~Birds that weigh more than 50 pounds cannot fly. A kiwi cannot fly. So, a kiwi weighs more than 50 pounds.~~

 5. **Compare and Contrast** How is deductive reasoning different from inductive reasoning?

## A Practice

If possible, use the Law of Detachment to make a conclusion. If it is not possible to make a conclusion, tell why.

◀ See Problem 1.

6. If a doctor suspects her patient has a broken bone, then she should take an X-ray.  
Dr. Ngemba suspects Lilly has a broken arm.
7. If a rectangle has side lengths 3 cm and 4 cm, then it has area  $12 \text{ cm}^2$ .  
Rectangle  $ABCD$  has area  $12 \text{ cm}^2$ .
8. If three points are on the same line, then they are collinear.  
Points  $X$ ,  $Y$ , and  $Z$  are on line  $m$ .
9. If an angle is obtuse, then it is not acute.  
 $\angle XYZ$  is not obtuse.
10. If a student wants to go to college, then the student must study hard.  
Rashid wants to go to Pennsylvania State University.

If possible, use the Law of Syllogism to make a conclusion. If it is not possible to make a conclusion, tell why.

◀ See Problem 2.

**STEM** 11. **Ecology** If an animal is a Florida panther, then its scientific name is *Puma concolor coryi*.

If an animal is a *Puma concolor coryi*, then it is endangered.

12. If a whole number ends in 6, then it is divisible by 2.  
If a whole number ends in 4, then it is divisible by 2.
13. If a line intersects a segment at its midpoint, then the line bisects the segment.  
If a line bisects a segment, then it divides the segment into two congruent segments.
14. If you improve your vocabulary, then you will improve your score on a standardized test.  
If you read often, then you will improve your vocabulary.

Use the Law of Detachment and the Law of Syllogism to make conclusions from the following statements. If it is not possible to make a conclusion, tell why.

◀ See Problem 3.

15. If a mountain is the highest in Alaska, then it is the highest in the United States.  
If an Alaskan mountain is more than 20,300 ft high, then it is the highest in Alaska.  
Alaska's Mount McKinley is 20,320 ft high.
16. If you live in the Bronx, then you live in New York.  
Tracy lives in the Bronx.  
If you live in New York, then you live in the eleventh state to enter the Union.
17. If you are studying botany, then you are studying biology.  
If you are studying biology, then you are studying a science.  
Shanti is taking science this year.

- 18. Think About a Plan** If it is the night of your weekly basketball game, your family eats at your favorite restaurant. When your family eats at your favorite restaurant, you always get chicken fingers. If it is Tuesday, then it is the night of your weekly basketball game. How much do you pay for chicken fingers after your game? Use the specials board at the right to decide. Explain your reasoning.
- How can you reorder and rewrite the sentences to help you?
  - How can you use the Law of Syllogism to answer the question?



**Beverages** For Exercises 19–24, assume that the following statements are true.

- A. If Maria is drinking juice, then it is breakfast time.
- B. If it is lunchtime, then Kira is drinking milk and nothing else.
- C. If it is mealtime, then Curtis is drinking water and nothing else.
- D. If it is breakfast time, then Julio is drinking juice and nothing else.
- E. Maria is drinking juice.

Use only the information given above. For each statement, write *must be true*, *may be true*, or *is not true*. Explain your reasoning.

- 19. Julio is drinking juice.
- 20. Curtis is drinking water.
- 21. Kira is drinking milk.
- 22. Curtis is drinking juice.
- 23. Maria is drinking water.
- 24. Julio is drinking milk.

- STEM 25. Physics** Quarks are subatomic particles identified by electric charge and rest energy. The table shows how to categorize quarks by their flavors. Show how the Law of Detachment and the table are used to identify the flavor of a quark with a charge of  $-\frac{1}{3}e$  and rest energy  $540\text{ MeV}$ .

Rest Energy and Charge of Quarks						
Rest Energy (MeV)	360	360	1500	540	173,000	5000
Electric Charge (e)	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$
Flavor	Up	Down	Charmed	Strange	Top	Bottom

Write the first statement as a conditional. If possible, use the Law of Detachment to make a conclusion. If it is not possible to make a conclusion, tell why.

- 26. All national parks are interesting.  
Mammoth Cave is a national park.
- 27. All squares are rectangles.  
 $ABCD$  is a square.
- 28. The temperature is always above  $32^\circ\text{F}$  in Key West, Florida.  
The temperature is  $62^\circ\text{F}$ .
- 29. Every high school student likes art.  
Ling likes art.
- 30. Writing** Give an example of a rule used in your school that could be written as a conditional. Explain how the Law of Detachment is used in applying that rule.



- 31. Reasoning** Use the following algorithm: Choose an integer. Multiply the integer by 3. Add 6 to the product. Divide the sum by 3.
- Complete the algorithm for four different integers. Look for a pattern in the chosen integers and in the corresponding answers. Make a conjecture that relates the chosen integers to the answers.
  - Let the variable  $x$  represent the chosen integer. Apply the algorithm to  $x$ . Simplify the resulting expression.
  - How does your answer to part (b) confirm your conjecture in part (a)? Describe how inductive and deductive reasoning are exhibited in parts (a) and (b).

**STEM 32. Biology** Consider the following given statements and conclusion.

**Given:** If an animal is a fish, then it has gills.

A turtle does not have gills.

**You conclude:** A turtle is not a fish.

- Make a Venn diagram to illustrate the given information.
- Use the Venn diagram to help explain why the argument uses good reasoning.



## Apply What You've Learned



Look back at the information about 2-by-2 calendar squares on page 81. Choose from the following words and expressions to complete the sentences below.

1 more than	7 more than	$a - 1$	$a - 7$	$d - 7$
1 less than	7 less than	$a + 1$	$a + 7$	$d - 1$
Syllogism	Detachment	$c + 1$	$c - 1$	$b + 7$

- Fact 1:** A date in a monthly calendar is ? a date directly to the left of it.
- Fact 2:** A date in a monthly calendar is ? a date directly above it.

Let  $a$ ,  $b$ ,  $c$  and  $d$  represent the numbers in a 2-by-2 calendar square, with  $a$ ,  $b$ ,  $c$ ,  $d$  listed from least to greatest.

- You can use Fact 1 above to conclude that  $b$  is equivalent to ?.
- You can use Fact 2 above to conclude that  $c$  is equivalent to ?.
- You can use Fact 1 above to conclude that  $d$  is equivalent to ?.
- Each of these conclusions is an example of using the Law of ?.